Compressibility Correction for Flow About Wing Surfaces

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Abstract

A COMPRESSIBILITY correction for the flow about external surfaces, such as wings, is necessary whenever the analysis of the flowfield is based on an incompressible technique This synoptic presents a modified technique that functions as a transformation between the compressible velocity field and an equivalent incompressible velocity field A set of fundamental incompressible solutions is combined to form a solution of interest which is corrected for compressibility using the Lieblein Stockman correction in a form modified for external flow This complete process is called the combination technique The results of this analysis are compared to other analytic results and experimental data This technique represents a simple method to obtain accurate subsonic flow solutions for two and three-dimensional geometries

Nomenclature

C	= chord
C_p	= pressure coefficient
$C_p{}^*$	= critical pressure coefficient
M_∞	= freestream Mach number
V_c	= velocity corrected for compressibility
V_i	= imcompressible velocity
$\bar{V}_i$	= average incompressible velocity
$V_{\infty i}$	= freestream incompressible velocity
x	= axial location
α	= angle of attack
η	= fractional spanwise location
ρ	= density
$\bar{\rho}_c$	= average compressible density
ρ_t	= total or incompressible density

Contents

Despite recent advances in computer codes, the analysis of a compressible flowfield about a general three dimensional body remains a formidable task. However, the analysis of an incompressible potential flowfield is more straightforward due to the linearity of the governing equation Because of this linearity superposition may be used If the flow is compressible, the principle of superposition of solutions cannot be used However, the effect of compressibility can be approximated by applying a compressibility rule or correction to an incompressible potential flow solution

The traditional methods of approximating the effect due to compressibility assume that the body being analyzed is thin, and a geometric transformation to the body is applied prior to analysis The Prandtl Glauert equation, which is widely used, is an example of a geometry modification technique Numerous compressibility rules or corrections exist, and nearly all of them require a geometry transformation Reference 1 shows the incompressible and compressible pressure distributions for an airfoil section having a thickness to chord ratio of 0 1681 and compares the results of various compressibility corrections Due to the thickness of the airfoil, the accuracy of the techniques leaves much to be desired Apart from any questions of accuracy most compressibility rules preclude the use of superposition of solutions because a geometry transformation is required Therefore, a compressibility correction which does not use a geometry transformation is needed for a technique which linearly combines solutions

The compressibility correction presented herein may be used in conjunction with any incompressible analysis and is especially applicable to a combination technique The combination technique is a method by which incompressible fundamental solutions are linearly combined to yield other incompressible solutions The technique is complete once use is made of a compressibility correction which transforms the incompressible velocity field into a compressible velocity field A compressibility correction of this type is the Lieblein Stockman correction[2] which has been used primarily for the analysis of internal flow The present work will show that the combination technique can be applied to compressible flow about external surfaces if a modified form of the Lieblein Stockman correction is used The Lieblein Stockman correction has the form

$$V_c = V_i (\rho_t / \bar{\rho}_c)^{V_i/\bar{V}_i} \tag{1}$$

where V_c is the compressible velocity V_i the incompressible velocity, $\bar{V}_i$ the average incompressible velocity, and $\bar{\rho}_c$ the average compressible density The stagnation density ρ_t is equal to the constant incompressible density Experience has shown that Eq (1) gives a velocity for locally supersonic flow which is too high (compared to experimental data) and

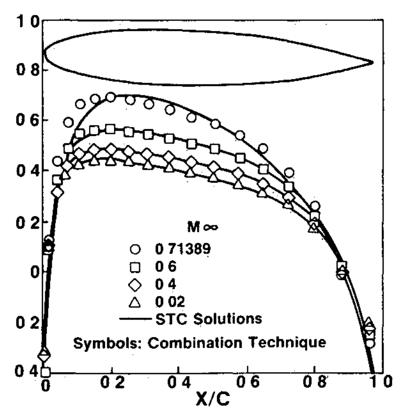

Fig 1 Comparison of the present technique with other analytic results for a symmetrical airfoil; $\alpha = 0$ deg

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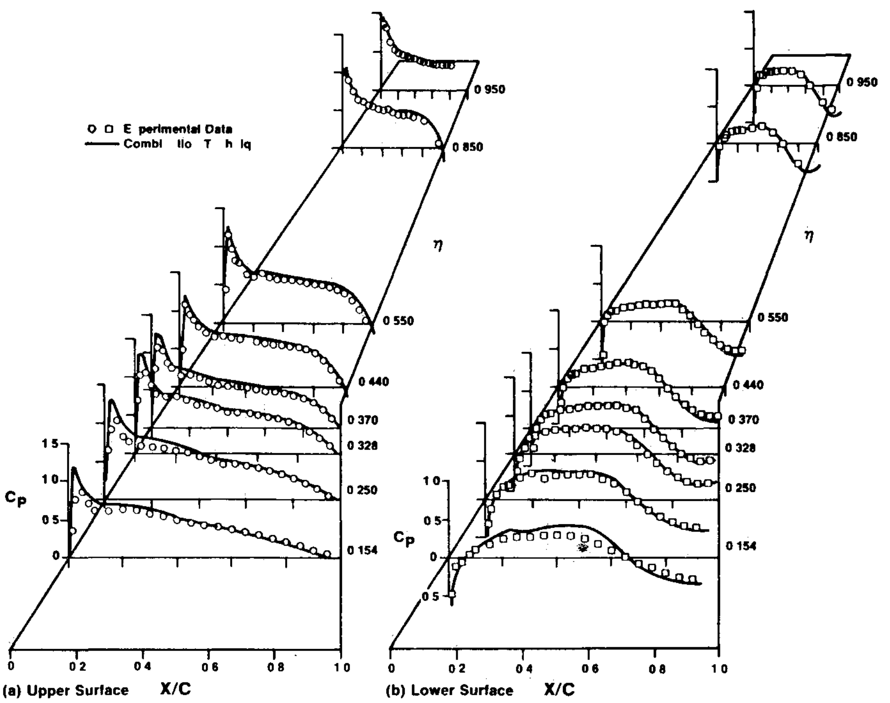

Fig 2 Comparison of the present technique with experimental data for a three dimensional wing; $M_\infty = 0\ 7$, $\alpha = 1\ 07$ deg

therefore an additional correction has been developed for supersonic regions The supersonic correction and detailed description of the complete combination technique is presented in Ref 3

The Lieblein Stockman correction has been shown to be very effective over a wide range of applications and con ditions for internal flow However, the correction has not been used extensively for external flow since it is not clear what the average incompressible velocity $\bar{V}_i$ of Eq (1) should be When the correction has been used for external flow calculations such as the external surfaces of inlets the average incompressible velocity has been taken to be the incompressible freestream velocity The present synoptic proposes an improved external flow correction and considers a two dimensional airfoil and a three dimensional wing It has been found that the average incompressible velocity can be equated to a weighted average of the freestream in compressible velocity and the local incompressible velocity, i e

$$\bar{V}_i = (\rho/\rho_t)_\infty V_{\infty i} + [1-(\rho/\rho_t)_\infty] V_i \qquad (2)$$

The weighting factor is the freestream density ratio and is a function of the freestream Mach number The relationship for the average incompressible velocity has been developed by comparing combination technique results and analytic results of a compressible flow program for a symmetric two dimensional airfoil

The procedure used to apply the compressibility correction is very straightforward since detailed information about the velocity field is not required The average incompressible velocity is used to determine the density ratio $\rho_t/\bar{\rho}_c$ in the same manner as Ref 2 Finally Eq (1) is used to calculate the compressible velocity V_c, and if the flow is locally supersonic the additional supersonic correction may be used In some instances however the calculated average incompressible velocity can be larger than the critical value For these cases, the density ratio $\rho_t/\bar{\rho}_c$ is set to the critical value and again V_c is calculated from Eq (1)

A two dimensional compressible flow analysis has been performed on a symmetrical airfoil section This airfoil has been selected since the results of applying various com pressibility rules to it are well documented in Ref 1 The analytical results for compressible flow have been used as reference solutions for the combination technique The comparisons of the analytic pressure coefficient data for various freestream Mach numbers are shown in Fig 1 The combination technique results are indicated by the symbols and the airfoil cross section is shown As may be seen in Fig 1, the agreement between the two techniques is very good especially when the highest freestream Mach number results are compared to the data shown in Ref 1 The poor agreement in Ref 1 between the compressible solution and the results of other compressibility rules is probably due to the thickness of the airfoil

The present technique has also been applied to a fully three dimensional geometry in the form of a wing fuselage model The paneled model that represents a NASA Langley research transport model is shown in Ref 3 Figure 2 shows the comparisons of the analysis and experimental data over a large extent of the wing span for a freestream Mach number of 0 7 This figure illustrates that the agreement is very good and the compressibility correction can be applied in a uniform manner at all locations

The compressibility correction presented in this work is a modification of the Lieblein Stockman correction which has been extensively applied to internal flow The modification of the correction for external flow uses a weighted average of the freestream incompressible velocity and the local in compressible velocity as the average incompressible velocity The compressibility correction, which does not modify the geometry, acts as a transformation between the desired compressible flowfield and an equivalent incompressible flowfield The analytic results have been shown to agree well with other analytic results and experimental data Provided the flow is subcritical or has only small regions of super critical flow the results are accurate over a wide range of applications

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